ANNEX 75

Time Series

Time series are rows of numbers that describe the development of social phenomena in time period.

Time series are composed of two main elements:
- the period of time (moment);
- the level of development of studied phenomena.

Time series can be double in nature. They can describe the size of phenomena in a period of time or in a moment of time. First called - the interval time series, the second - the moment time series.

Depending on quantities express the levels of time series :
- the time series of absolute quantities;
- the time series of relative quantities;
- the time series of average quantities.

1. Indicators of absolute and relative change of Time series levels

1) **Absolute increase** – characterize the increase/ decrease the level of row in concrete time period

a) chain absolute increase/decrease:

$$\Delta m(ch) = Y_m - Y_{m-1}$$

b) basic absolute increase/decrease:

$$\Delta m(b) = Y_m - Y_1$$

where:

Y_m - any level of time series

Y_{m-1} - previous level of time series

Y_n - the last level of time series

Y_1 - initial (first) level of time series

Between the chain and the base absolute increase (decrease) is as follows the mathematical relationship: the sum of chain absolute increase (decrease) equal to the base absolute increases (decreases) corresponding to the last level of the time series:

$$\sum_{m-1}^{n} \Delta m(ch) = \Delta n(b)$$

2) **Growth rate** – indicator of intensity of changes of time series levels, which shows the speed of development of studied phenomenon

a) chain growth rate:

$$T_{m(ch)} = \frac{Y_m}{Y_{m-1}}$$

b) basic growth rate:

$$T_{m(b)} = \frac{Y_m}{Y_1}$$

Between the chain and the base growth rates) is as follows the mathematical relationship: the multiplication of chain growth rates equal to the corresponding base growth rate:

$$T_{1(ch)} * T_{2(ch)} \ldots T_{n(ch)} = T_{n(b)}$$

3) **Increase rate** – show for how many percentages the level of the corresponding row in comparing with the previous level

a) chain increase rate:

$$t_{m(ch)} = \frac{\Delta_{m(ch)}}{Y_{m-1}} * 100$$

b) basic increase rate:

$$t_{m(b)} = \frac{\Delta_{m(b)}}{Y_1} * 100$$

4) **The absolute meaning of increase/decrease by 1% -** express the real matter of increase/decrease rate:

$$t_{m(1\%)} = \frac{\Delta_{m(ch)}}{t_{m(ch)}}$$

or

$$t_{m(1\%)} = \frac{Y_n - Y_1}{(Y_n : Y_1) * 100 - 100}$$

2. The average quantities of time series

1) for the interval time series **the average level** is calculated:

$$\overline{Y} = \frac{\sum_{m=1}^{n} Y_m}{n} = \frac{Y_1 + Y_2 + Y_3 + \ldots + Y_n}{n}$$

2) for the moment time series (if the same lengths of time periods are between levels) **the average level** is calculated:

$$\overline{Y} = \frac{0{,}5Y_1 + Y_2 + Y_3 + \ldots + 0{,}5Y_n}{n-1}$$

where: n-1 – number of moments

3) for the moment time series (if the different lengths of time periods are between levels) **the average level** is calculated:

$$\overline{Y} = \frac{(Y_1 + Y_2)l_1 + (Y_2 + Y_3)l_2 \ldots + (Y_{n-1} + Y_n)l_{n-1}}{2 * (l_1 + l_2 + \ldots l_{n-1})}$$

where: $\overline{Y}$ – levels of time series,
l_1 - time periods

4) **average absolute increase/decrease** :

$$\overline{\Delta}_{(ch)} = \frac{\sum_{m=1}^{n} \Delta_{m(ch)}}{n_\Delta}$$

where: n_Δ – number of absolute increases

or

$$\overline{\Delta}_{(b)} = \frac{Y_n - Y_1}{n-1}$$

where: n-1– number of levels of row

5) **chain average growth rate** :

$$\overline{T}_{ch} = \sqrt[n]{T_{1(ch)} * T_{2(ch)} * \ldots * T_{n(ch)}} = \sqrt[n]{\prod_{m=1}^{n} T_{m(ch)}}$$

where: n– number growth rates of chain
T – individual growth rates of chain, expressed in coefficients

or

$$\overline{T}_b = \sqrt[n-1]{\frac{Y_n}{Y_1}}$$

6) **chain average increase rate** :

$$\bar{t} = \overline{T} - 100{,}0\%$$ (if expressed in %)

or

$$\bar{t} = \overline{T} - 1$$ (if expressed in coefficients)

3. Method of moving averages

The method is based on the range expansion principle, but it is not a calendar interval extension.

This method is particularly suitable for those cases where the phenomena of development is irregular, zigzag nature (highs and lows). Each time series moving average is calculated in the following order:

- determine smoothing interval, number of the levels in the range.
- meanings of average levels of smoothing interval are calculated, which at the same time are the smoothing meanings of time series.

To calculate uneven number (such as 3 members) of the moving average, in first calculate sum of levels of moving intervals, then divided by the number of members (levels) of intervals.

The results – moving averages relate to the members existed in the middle of interval. Interval latter members. 3 members moving averages van not been calculated for the first and the last empirical level of row.

To calculate even number (such as 4 members) of the moving average, in first calculate sum of levels of moving intervals, then divided by the number of members (levels) of intervals. Such moving averages are called as not centred moving averages and are placed between periods.

$$\overline{Y_1} = \frac{Y_1 + Y_2 + Y_3 + Y_n}{4}; \quad \overline{Y_2} = \frac{Y_1 + Y_2 + Y_3 + Y_n}{4} \text{ etc.}$$

4 members centred moving average is calculated as 2 members of the moving average of the two not centred averages:

$$\overline{Y_{1(c)}} = \frac{\overline{Y_1} + \overline{Y_2}}{2} \text{ etc.}$$

Such averages cannot been calculated for 4 empirical members – for 2 on the beginning of row and 2 of the end of row.

4. Method of analytic smoothing, if studied phenomena has regular character

$$\tilde{Y}_1 = a_0 + a_1 t$$

where

$\tilde{Y}_1$ - levels of smoothing row

a_0, a_1 - parameters of straight line
t- time indicators

For the simplified of system – we can move beginning of time to middle of row. If row consist of uneven numbers, than “0 “refers to central level, “-1” refer to one level before etc., and 1, 2 etc. refer to levels after.
In this case:

$$\Sigma_t = o$$

$$a_0 = \frac{\sum Y_i}{n}$$

$$a1 = \frac{\sum Y_i t}{\sum t^2}$$

If number of levels of time series is even quantity, then for reaching $\Sigma_t = o$, we have to expand (double) the measure of time order of levels and meaning of coefficient **t** to start with 1-n

5. Indexes of seasonality

Arithmetical average method:

Individual indexes:

$$i_s = \frac{Y_m}{\overline{\overline{Y}}} * 100$$

where
Y_m - the actual level of unit in year m
$\overline{Y}$ - the average level of time unit (month, quarter) per year

$$\overline{Y} = \frac{\sum_{m=1}^{n} Y_m}{n}$$

Where n-m – number of time units (months, quarters) per year

$$\overline{i_s} = \frac{\overline{Y_m}}{\overline{\overline{Y}}} * 100$$

where

$\overline{Y_m}$ - the average level of month or quarter calculated from the data per more than 1 years

$\overline{\overline{Y}}$ - the average level of time unit in all years period, calculated from the data per more than 1 years

Average quadratic error of index of seasonality:

$$\sigma_s = \frac{\sqrt{\sum (i_s - 100)^2}}{n}$$

Where i_s- index of seasonality of month, quarter