ANNEX 77

FINANCE MANAGEMENT

(Working material)

Chief Actuary

Prof. Gaida Pettere

"BTA INSURANCE COMPANY" SE

FUNDAMENTALS of INVESTMENT

I THEORY OF INTEREST RATES

1.1 ACCUMULATION

Interest may be regarded as a reward paid by one person or organization (the borrower) for the use of an asset, referred to as capital, belonging to another person or organization (the lender).

When expressed in monetary terms, capital is also referred to as *principal*.

Example 1.1

An in investor who had opened an account some time ago with an initial deposit of LVL 100, and who have made no other payments to or from the account, would expect to withdraw more than LVL 100 if he were now to close the account. Suppose, for example, that he receives LVL 106 on closing his account.

We shall say LVL 100 – initial deposit (principal)

LVL 6 - interest.

Interest usually expressed in percentages $\frac{6}{100} \cdot 100 = 6\%$.

The interest is a payment by the borrower to the investor for the use of his capital.

We shall denote interest by i expressed in decimal fraction and by C initial capital or principal.

Example 1.2

Suppose you invest LVL 100 in a bank account that pays 8% interest per year. What shall you receive after year?

Always the precise conditions of any transaction will be mutually agreed.

For example:

a) after stated period the capital may be returned to the lender with the interest due,
b) several interest payments may be made before the asset is finally returned to the borrower.

Simple interest:

Under a simple interest rule, money invested for a period different from 1 year accumulates interest proportional to the total time of the investment.

$$A = C \cdot (1 + t \cdot i)$$

Example 1.3

Suppose LVL 900 is deposited in a savings account, which pays simple interest at the rate of 5% per annum. Assuming that there are no subsequent payments to or from the account, find the amount finally withdrawn if the account is closed after a) six months, b) ten months, c) one year, d) three years.

Compound interest:

If interest is compounded yearly, then after 1 year, the first year's interest is added to the original principal to define the larger principal base for the second year. Thus during the second year, the accounr earns interest on interest. This is the compounding effect, which is continued year after year.

Accumulation under the compound interest:

$$\begin{aligned}
&A(1) = C + C \cdot i = C \cdot (1+i) \\
&A(2) = A(1) + A(1) \cdot i = A(1) \cdot (1+i) = C \cdot (1+i)^2 \\
&A(3) = C \cdot (1+i)^3 \\
&\ldots \\
&A(t) = C \cdot (1+i)^t
\end{aligned}$$

Example 1.5

Suppose LVL 900 is deposited in a savings account, which pays compound interest at the rate of 5% per annum. Assuming that there are no subsequent payments to or from the account, find the amount finally withdrawn if the account is closed after a) six months, b) ten months, c) one year, d) three years.

The seven-ten rule

Money invested at 7% per year doubles in approximately 10 years. Also, money invested at 10% per year doubles in approximately 7 years.

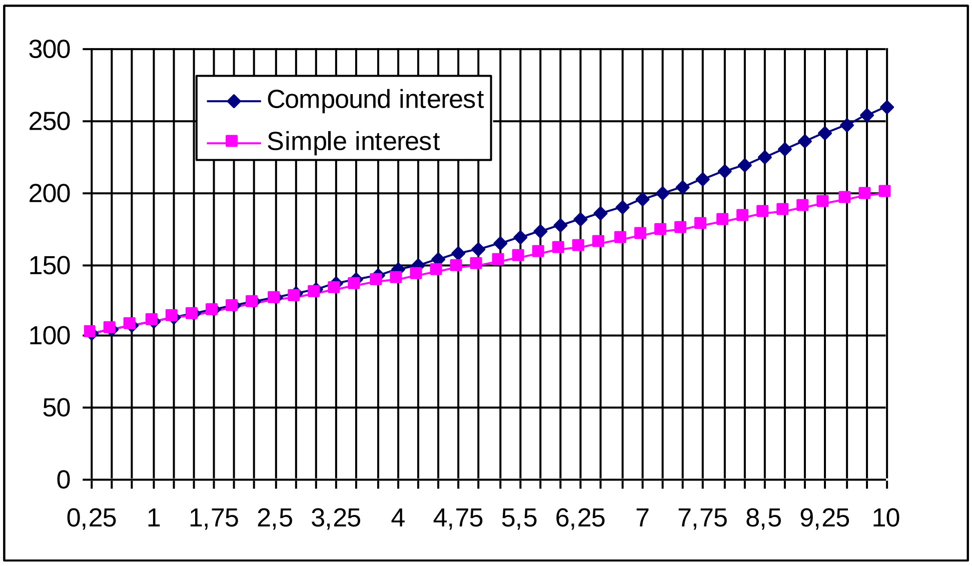


Figure 1. Accumulation of LVL 100 by interest 10%.

If interest is compounded once per year the compound interest rate is called **effective rate of interest**.

Often interest is paid more frequently like once per year: credits, deposits, and bonds.

The annual rate of interest compounded more frequently like once per year is called **nominal rate of interest**.

Nominal rate of interest per unit of time on transaction is such that the effective rate of interest for the period of length h is $h \cdot i_h$.

We shall consider only situation when $h = \frac{1}{p}$ where p is 4 for quarter, 2 for half year and 12 for month, 365 for day and we shall denote nominal rate of interest $i^{(p)}$.

$$A(t) = \left(1 + \frac{i}{p}\right)^t$$

For example, an annual rate of 6% compounded every half year produce income in year

in month

in two years

Example 1.7

Suppose LVL 900 is deposited in a savings account, which pays annual interest rate of 8% compounded quarterly. Assuming that there are no subsequent payments to or from the account, find the amount finally withdrawn if the account is closed after a) six months, b) nine months, c) one year, d) three years.

Money invested today leads to increased value in the future as a result of interest. The formulas of the previous section show how to determine **future value**. This concept can be reversed in time to calculate the value that should be assigned now, in the present, to money that is to be received at a later time. This value of money is called **present value.**

1.2 PRESENT VALUE

Example 1.8

How much shall we need to put in deposit now to receive after year LVL 100, if bank guarantees 4% effective annual interest rate?

Example 1.9

How much shall we need to put in deposit now to receive after two years LVL 100, if bank guarantees 4% effective annual interest rate?

$$PV = \frac{A}{(1+i)^t}$$

Example 1.11

How much shall we need to put in deposit now to receive after twenty days LVL 100, if bank guarantees 4% effective annual interest rate (year-365 days)?

1.3 FUTURE AND PRESENT VALUES OF PAYMENT STREAMS

Let us assume that cash arrives or we need to pay many times after fixed period, which can be year, quarter, month or some another period. Every payment can be different and some of them can be zero. Cash flows can arrive or have to be paid in the beginning of period or in the end. Such cash flows are called streams.

Future value of the stream Given a cash flow stream $(x_1,...,x_n)$ and interest rate i each period. Find the future value of the stream.

$$A(n) = x_1 \cdot (1+i)^n + x_2 \cdot (1+i)^{n-1} + ... + x_n \cdot (1+i)$$

or

$$A(n) = x_1 \cdot (1+i)^{n-1} + x_2 \cdot (1+i)^{n-2} + ... + x_n$$

The present value of a general cash flow stream – like future value – can also be calculated by considering each flow element separately.

Present value of a stream Given a cash flow stream $(x_1,...,x_n)$ and interest rate i each period. Find the present value of the stream.

$$PV = x_1 + \frac{x_2}{1+i} + \frac{x_3}{(1+i)^2} + \frac{x_4}{(1+i)^3} + ... + \frac{x_n}{(1+i)^{n-1}}$$

or

$$PV = \frac{x_1}{1+i} + \frac{x_2}{(1+i)^2} + \frac{x_3}{(1+i)^3} + ... + \frac{x_n}{(1+i)^n}$$

Example 1.13

Consider a cash flow stream $(100, 50, 30, 0, 100)$ when the periods are years and the annual interest rate is 8% effective.

Calculate the future value a) after 5 years if payments are in the beginning of each year,

b) after 5 years if payments are in the end of each year.

Example 1.16

Consider a cash flow stream $(100, 50, 30, 0, 100)$ when the periods are years and the annual interest rate is 8% effective.

Calculate the present value a) if payments are in the beginning of each year,

b) if payments are in the end of each year.

2 BASIC COMPOUND INTEREST FUNCTIONS

Series of n payments, each of amount 1, to be made at time intervals of one unit are called **annuities-certain** (or simply annuities).

If first payment is made after one unit of time annuity is called **annuity in arrear** or immediate annuity.

If first payment is made in advance (in moment 0) annuity is called **annuity-due**.

Present values

One unit of time before the first payment is made (annuity in arrear).

$$a_{\overline{n|}} = \frac{1 - \frac{1}{(1+i)^n}}{i}$$

At the time the first payment is made (annuity-due):

$$\ddot{a}_{\overline{n|}} = \frac{1 - \frac{1}{(1+i)^n}}{i} \cdot (1+i)$$

Futures value

At the time the last payment is made (annuity in arrear).

$$s_{\overline{n|}} = \frac{(1+i)^n - 1}{i}$$

One unit of time after the last payment is made (annuity-due).

$$\ddot{s}_{\overline{n|}} = \frac{(1+i)^n - 1}{i} \cdot (1+i)$$

Example 2.1

A loan of LVL 3000 is to be repaid by 12 equal annual instalments. The rate of interest for the transaction is 6% per annum effective. Find the amount of each annual repayment, assuming that payments are made (a) in arrear and (b) in advance.

Example 2.2

A loan of LVL 5000 is to be repaid in three years by monthly payments. The rate of interest for the transaction is 6% per annum effective. Find the amount of each annual repayment, assuming that payments are made (a) in arrear and (b) in advance.

Example 2.3

Investor has agreed to pay LVL 20 each month in a saving account 5 years (60 payments). Bank is using annual interest rate 4% effective for its saving accounts. Find the accumulated sum of investor after 5 years from now if he is going to start his payments (a) from now in the beginning of each month, (b) from now in the end of each month.

3 DISCOUNTED CASH FLOW (NET PRESENT VALUE AND YIELDS)

3.1 INTERNAL RATE OF RETURN

Example 3.1

Given cash flow stream $(-3,1,1,3)$ with time interval one year and payments in the beginning of each year. Write the formula for PV of that stream and sketch the graph showing relations between PV and effective annual interest rate.

Interest rate (i)	0	0.02	0.04	0.06	0.08	0.10	0.12
PV							

Formal definition

Let $(x_0, x_1, x_2, ..., x_n)$ be a cash flow stream. Then the **internal rate of return** (or **yield** of transaction) of this stream is a number r satisfying the equation

$$0 = x_0 + \frac{x_1}{1+r} + \frac{x_2}{(1+r)^2} + ... + \frac{x_n}{(1+r)^n} .$$

Equivalently, it is a number r satisfying

$$\frac{1}{1+r} = v \quad \text{or} \, r = \frac{1}{v} - 1 \quad ,$$

where v satisfies the polynomial equation

$$0 = x_0 + x_1 \cdot v + x_2 \cdot v^2 + ... + x_n \cdot v^n .$$

Main theorem of internal rate of return

Suppose the cash flow stream $(x_0, x_1, x_2, ..., x_n)$ has $x_0 < 0$ and $x_k \geq 0$ for all k, $k = 1, 2, ..., n$ with at least one term being strictly positive. Then there is a unique positive root to the equation

$$0 = x_0 + x_1 \cdot v + x_2 \cdot v^2 + ... + x_n \cdot v^n .$$

Furtermore, if $\sum_{k=0}^{n} x_k > 0$ (meanings that the total amount returned exceeds the initial investment), then the corresponding internal rate of return $r = \frac{1}{v} - 1$ is positive.

3.2 EVALUATION CRITERIA

The investor always is before the problem in which investment project is better to invest. Therefore alternative possible cash flow streams must be evaluated according to a logical and standard criterion. Several different criteria are used in practise, but the two most important methods are based on present value and on internal rate of return.

3.2.1 NET PRESENT VALUE

The investment or project will normally require an initial outlay and possible other outlays in future, which can be positive and negative. These cash flows may be completely fixed or they may have to be estimated. Therefore we shall denote by

$$x_k = ci_k - co_k$$

where ci_k is cash inflow in time k and co_k is cash outflow in time k. To emphasize that, the expression net present value (NPV) is used.

Example 3.2 (Which onions to buy?)

Suppose that you have the opportunity to buy and plant onions, which are blossoming after year and another ones, which are blossoming after two years but with most expensive flowers. Onions cannot be used more after cutting flowers. Both projects requires an initial outlay of money to purchase and plant onions. No other cash flows occur until flowers are cut and sold. We assume that the cash flow streams with these two projects are $(-100, 300)$ and $(-100, 0, 400)$. We also assume that interest rate is 7%.

Calculate net present value for both projects and compare them.

3.2.2 INTERNAL RATE OF RETURN

Internal rate of return can also be used to evaluate investment projects. The rule is: the higher internal rate of return, the better investment projects.

Example 3.3 (Which onions to buy, continued)

Evaluate both projects from example 3.2 to use internal rate of return approach.

Approximate formula for calculation of internal rate of return

$$r = \frac{\bar{I} + \frac{P_n - P_0}{n}}{\frac{P_n + P_0}{2}}$$

where P_0 is the price of the asset in the beginning, P_n is the price of the asset at time n (after n years), $\bar{I}$ is the average yearly income received during n years.

3.3 APPLICATIONS AND EXTENSIONS

Exercise 3.4 (Cost evaluation)

Suppose you have to buy for office computer technique and you have to choose between two possibilities. Technique A cost LVL 4000 and expected to have maintenance cost LVL 1000 per year in the end of each year of using and assumed has useful life 4 years without salvage value. Technique B cost LVL 6500 and expected to have maintenance cost LVL 1000 per year in the end of each year of using and assumed has useful life 6 years without salvage value. The interest rate is 7%.

Prepare one cycle and combined cycle analysis for twelve years.

INFLATION

Inflation is characterized by an increase in general prices with time. It can be described quantitatively in terms of an inflation rate f .

That leads us to define the real interest rate. It shows how much money increase in reality taking account an inflation:

$$1 + i_0 = \frac{1 + i}{1 + f} \quad \text{or} \quad i_0 = \frac{i - f}{1 + f}$$

Example 3.5

Suppose that inflation is 6%, the annual interest rate 10%, and we have estimated cash flow of our business in real LVL as shown in the second column of table.

Calculate the present value in real terms.

Year	Cash flow (LVL)
0	-20000
1	7000
2	7000
3	7000
4	4000

4 RISK AND RETURN

4.1 THE MARKET ENVIRONMENT AND SECURITIES

BONDS

A **bond** is a financial contract. The bond issuer will pay the bond's buyer periodic interest and at the end of the specified term the principal, also called par value.

Bond characteristics

Principal — Coupon payments

Sum at maturity — Fixed periodic interest instalments

Example 4.1

Sketch the diagram of the cash flow of a 4%, semi annual, three-year bond with par value LVL 1000.

Advantages	Disadvantages
Good sources of current income.	Potential profit is limited.
Relatively safe from large losses	Profit is very sensitive to inflation.
Coupon payments are paid before dividends	Profit is very sensitive to interest rates.

STOCKS

COMMON STOCK

Common stock (common shares or equity) represents part ownership in a firm.
Owner of common stock has rights:

1) has rights to take part in managing of firm,
2) to receive part from profit what is left over after all other claims,
3) to receive part of salvage value of firm after all another claims are satisfied in the case of bankruptcy.

Cash dividends are part of profit paid to stockholder after other liabilities have been paid.

Declaration date is the day when the board of directors actually announces that stockholders on the date of record will receive dividend.

Date of record is the day on which the stockholder must actually own the shares to be able to receive the dividend.

Ex-dividend date is the first day on which, if the stock is purchased, stockholder are no longer allowed to receive the dividend.

The payment day is the day that the company actually pays dividends.

(For example, in NYSE ex-dividend date is four trading days before the date of record. The payment day is about three weeks after the ex-dividend date).

Common stock does not have a date on which the cooperation must buy them back.

OTHER SECURITIES

Preferred stocks, Forwards, Futures, Swaps, Puts, Calls, … .

4.2 RATE OF RETURN

Why the rate of return is needed?

Example 4.2

Financial information in LVL about four investments is shown in following table. Which investment project is the best?

	Deposit	Stock	Bond	Land
Income in I quarter	15	10	0	0
Income in II quarter	15	10	50	0
Income in III quarter	15	10	0	0
Income in IV quarter	15	15	50	0
Beginning of year	1000	2000	1000	3000
End of year	1000	2100	970	3200
Income per year				
Capital gain or loss				
Total income				
HPR (%)				

The past rates of return are used for several purposes.

1. Measuring historical performance (is possible to compare the rates of return on alternative investments).
2. Estimating future rates of return.
3. Estimating cost of capital.

4.2.1 SIMPLE RATE OF RETURN (HPR)

Investors hold security for a given period. The rate of return measured for this period is called the holding period return (HPR).

The simple rate of return measures how much the value of a given investment increases or decreases over a given period of time.

$$r_t = \frac{P_t - P_{t-1} + I_t}{P_{t-1}}$$

where P_{t-1} is the price of the asset at time $t-1$, P_t is the price of the asset at time t, I_t is the income paid at time t. This formula fit for periods of length one year or less.

Example 4.3

Calculate HPR to use the simple rate of return for investment projects in exercise 4.2.

Example 4.4

Let assume you purchased one share of stock at the beginning of the year for LVL 100 per share and you have received LVL 6 cash dividend per share in the end of year. The price of stock in the end of year is LVL 105 per share. Calculate rate of return if you are going to sell having by you share of stock.

Example 4.5

Prices and dividends received in each corresponding year are shown in following table. Calculate HPR in each year assuming that dividends are not invested further and they are received in the end of year.

Year	Dividends (LVL)	Price in market (LVL)		HPR (%)
		Beginning of year	End of year	
1	2	3	4	5
2000	2	90	95	
2001	4		100	
2002	5		110	
2003	5		100	
2004	4		115	

Simple rate of return formula is created for situations when income is received at the end of the period. If income is paid during the period the simple rate of return has certain limitations.

If cash flows occurs during the period, they must be theoretically be used to buy additional units of the investment (reinvested). Then the most accurate approach is to calculate simple rate of return for the sub-period, and then link sub-period returns to get the return for month, quarter, semi-annual or annual to use formula:

$$r = \left[(1+r_1)\cdot(1+r_2)\cdot\ldots\cdot(1+r_n)\right]-1.$$

It is called linking method.

If the holding period is longer than one year the annual rate of return is calculated as internal rate of return of investment:

$$r = \frac{\overline{I} + \frac{P_k - P_0}{n}}{\frac{P_k + P_0}{2}}$$

where P_0 is the price of the asset in the beginning, P_k is the price of the asset at time k (time when investor plans to sell asset), $\overline{I}$ is the average yearly income received during k years.

Example 4.6

Investor plans after investing LVL 10000 in new business to receive three years in the end of each year LVL 2000 but in the end of fourth year after selling business LVL 8000.

Write formula for PV calculation and calculate Internal rate of return for that project.

4.2.2 AVERAGE RATE OF RETURN: THE MEASURE OF PROFITABILITY

The past average rate of return measures the average profitability of an investment.

Example 4.7

Suppose that you observe the rates of return on two stocks. They are shown in the table.

Year	Rate of return (%)	
	Stock A	Stock B
1	-20	2
2	50	24

Which stock has a better historical return?

Arithmetic average:

$$\bar{r}_i = \frac{\sum_{t=1}^{n} r_{i,t}}{n}$$

Geometric mean:

$$\bar{r}_{i,g} = \left[(1+r_{i,1})\cdot(1+r_{i,2})\cdot\ldots\cdot(1+r_{i,n})\right]^{\frac{1}{n}} - 1$$

Example 4.7

Calculate arithmetic and geometric mean for rates of return for stocks in example 4.6.

Which one is better?

Example 4.8

Suppose a unit trust began with the LVL 100 per 100 units. At the end of first year cost of 100 units were LVL 50, but in the end of second year once more LVL 100.

Calculate arithmetic and geometric mean of return and compare with real return.

Example 4.9

Calculate the arithmetic and geometric annual average rates of return for the two investment funds. Explain the difference between the two results.

Year	Rate of return (%)	
	Fund A	Fund B
1	15	10
2	5	14
3	-10	8
4	50	12
Arithmetic mean		
Geometric mean		

4.3 RISKS OR BONDS AND STOCKS

1. Default risk

Company can fail to pay coupon or nominal if it has bonds or can have bankruptcy in case of stocks.

2. Interest rate risk

Change in interest rates affects prices of bonds and stocks.

3. Inflation rate risk

Inflation rate risk or purchasing power risk affects purchasing power of future cash receipts.

4. Risk of call

Many bonds contain call provisions. Risk that bond will be taken back if interest rates goes down.

5. Liquidity risk

Risk that investor will not be able sell security at all or to receive appropriate price.

6. Political and regulatory risk

Bonds and stocks are also exposed to political and regulatory risk. This risk refers to unforeseen changes in the tax or legal environment that have impact on prices.

7. Business risk

Stocks and corporate bond prices are influenced greatly by the prosperity of the particular company, as well as by economy in general.

8. Market risk

The prices of all securities in a particular market tend to move together.

9. Exchange rate risk

Exchange rate risk is a risk of exchange rates if investment is in different currencies.

4.4 THE MEASURES OF RISK

Volatility of past rates of return also provides some information for future.

Example 4.10

Suppose that you observe the rates of return on two stocks. They are shown in the table.

Year	Rate of return (%)	
	Stock A	Stock B
1	-20	6
2	0	8
3	50	7

The dispersion of the past rates of return around the mean measures the historical volatility related to the corresponding investment (or risk).

The main measure of dispersion is variance or standard deviation.

Variance

$$\sigma_i^2 = \frac{\sum_{t=1}^{n}(r_{i,t} - \overline{r_i})^2}{n-1} .$$

Standard deviation

$$\sigma_i = \sqrt{\sigma_i^2} .$$

Example 4.11

Calculate the standard deviation of rates of return of stocks A and B in exercise 4.10.

Historical **risk premium**	=	Historical average rate of return	-	Historical riskless interest rate

Exercise 4.12

Calculate historical risk premium for stocks in example 4.10 assuming that historical riskless interest rate is 4% .

4.4 THE FUTURE

4.4.1 THE EXPECTED RATE OF RETURN

Example 4.13

Suppose you have two possibilities to invest for one year:

1) You can buy government bond with zero coupon rate and LVL 1000 face value. The price of bond is LVL 900. The bond matures exactly one year from today.

2) You can buy stock, which is traded by LVL 900. Suppose no dividends are paid, and the stock price at the end of the year is either LVL 1000 with a probability (chance) of ½ or LVL 800 with a probability (chance) of ½.

Calculate the rates of return for both investments.

Certainty is the situation in which the future value of the asset is known with probability 1.	**Uncertainty** or **risk** is the situation in which there is more than one possible future value of the asset. In such case we say that the future value is a **random variable**.

PROBABILITY

Objective probabilities	Subjective probabilities
Calculated from past expierence	Estimated probabilities

The expected rate of return:

$$E(R) = \sum_{i=1}^{n} p_i \cdot r_i$$

Example 4.14

Assume you have three possibilities to invest in different securities shown in following table. All investments require the same initial outlay LVL 100000.

Security A	Security B	Security C

Rate of return (%)	Probability	Rate of return (%)	Probability	Rate of return (%)	Probability
4	1	-4	¼	-8	¼
		0	¼	6	½
		20	½	40	¼

Calculate the expected rate of return for given securities. Suppose that you have to select the investment by the expected rate of return. Which is better? Do you agree with results?

4.4.2 THE MEASURES OF RISK (VARIANCE AND STANDARD DEVIATION)

One of the measures of dispersion around the mean is variance. It is often used like measure of risk (If there are no significant changes of rates of return in time).

Variance:

$$\sigma^2 = \sum_{i=1}^{n} p_i \cdot \left[r_i - E(R) \right]^2 .$$

Standard deviation:

$$\sigma_i = \sqrt{\sigma_i^2} .$$

Example 4.15

Calculate the variance with both methods and standard deviation for rates of return of securities given in example 4.14.

4.4.3 THE MEAN-VARIANCE CRITERION

In a comparison of two investments, A and B, there are six possibilities:

1. $E(R_A) > E(R_B)$ and $\sigma_A^2 < \sigma_B^2$.
2. $E(R_A) > E(R_B)$ and $\sigma_A^2 = \sigma_B^2$.

3. $E(R_A) > E(R_B)$ and $\sigma_A^2 > \sigma_B^2$.

4. $E(R_A) = E(R_B)$ and $\sigma_A^2 < \sigma_B^2$.

5. $E(R_A) = E(R_B)$ and $\sigma_A^2 = \sigma_B^2$.

6. $E(R_A) = E(R_B)$ and $\sigma_A^2 > \sigma_B^2$.

The Mean-Variance Criterion

The MVC is used to select those assets (or portfolios of assets) with

(1) the lowest variance for the same (or higher) expected return,

or

(2) the highest expected return for the same (or lower) variance.

Example 4.16

Choose the security in example 4.14 to use MVC.

Example 4.17

Assume you have three possibilities to invest in different securities shown in following table. All investments require the same initial outlay LVL 100.

Security A		Security B		Security C	
Return (LVL)	Probability	Return (LVL)	Probability	Return (LVL)	Probability
120	1	110	½	100	½
		130	½	140	½
Expected return		Expected return		Expected return	
Variance		Variance		Variance	

Calculate the expected return for given securities and variance of return. Suppose that you have to select the investment by the MVC. Which is better? Do you agree with results?

ATTITUDES TOWARDS RISK

- Investors who do not like risk (volatility) are called **risk averters**.
- Investors who are **risk neutral** completely ignore an asset's variance and make investment decisions based only on the asset's expected rate of return.
- Investors who like the risk (are able to higher price for an asset whose variance is larger) are called **risk seekers**.

5. BOND VALUATION

FIXED-INCOME SECURITIES

- Saving Deposits
- Money Market Instruments
- Bond
- Government bond: treasury bill, treasury bond
- Mortgage bond
- Preferred stock
- Convertible bond

Quality ratings

	Moody's	Standard & Poor's
High grade	Aaa	AAA
	Aa	AA

Medium grade	A	A
	Baa	BBB
Speculative grade	Ba	BB
	B	B
Default grade	Caa	CCC
	Ca	CC
	C	C
		D

The assignment of a rating class by a rating organization is largely based on the issuer's financial status as measured by various financial ratios. For example, the ratio of debt to equity, the ratio of current assets to current liabilities, the ratio of cash flow to outstanding debt, as well as several others are used.

Price of bond

The ***bid price*** is the price dealers are willing to pay for the bond. The ***ask price*** is the price at which dealers are willing to sell the bond, and hence the price at which it can be bought immediately.

Prices are quoted as a percentage of face value in the moment on next coupon payment date. Therefore bond quotations ignore **accrued interest**, which must be added to the price quoted in order to obtain the actual amount that must be paid for the bond.

Used notations:

N **-**face value,

R **-** redemption price per unit of nominal expressed as decimal number,

n - time to maturity expressed in years,

g - rate of coupon expressed as decimal number,

m - frequency of coupon payments in year,

i - desired rate of investment by investor ,

P - calculated price in the moment of next coupon payment,

AI - accrued interest

P_0 - bond price in existing moment,

P_k - estimated bond price after k years,

k -years after whic investor plans sell bond,

t_1 - number of days since the last coupon payment,

t_2 - number of days untill next coupon payment,

K - number of coupon payments from settlement data to maturity,

E - total number of days between two coupon payments

Bond price formula (called clean price):

$$P_0 = \frac{g \cdot N}{m} \cdot \frac{1 - \left(\frac{1}{1 + \frac{\lambda}{m}} \right)^{n \cdot m}}{\frac{\lambda}{m}} + R \cdot N \cdot \left(\frac{1}{1 + \frac{\lambda}{m}} \right)^{n \cdot m}$$

More precise calculations

$$P_0 = \frac{R \cdot N}{\left(1 + \frac{\lambda}{m}\right)^{K - 1 + \frac{t_2}{E}}} + \sum_{k=1}^{K} \frac{N \cdot \frac{g}{m}}{\left(1 + \frac{\lambda}{m}\right)^{k - 1 + \frac{t_2}{E}}} - N \cdot \frac{g}{m} \cdot \frac{t_1}{E}$$

Yield:

$$IRR = YTM = \lambda$$

Internal Rate of Return = Yield to Maturity

Therefore yield is root of equation:

$$P_0 = \frac{g \cdot N}{m} \cdot \frac{1 - \left(\frac{1}{1 + \frac{\lambda}{m}} \right)^{n \cdot m}}{\frac{\lambda}{m}} + R \cdot N \cdot \left(\frac{1}{1 + \frac{\lambda}{m}} \right)^{n \cdot m}$$

Approximately it can be calculated to use the formula:

$$\lambda \approx \frac{g \cdot N + \frac{R \cdot N - P_0}{n}}{\frac{R \cdot N + P_0}{2}} .$$

Another yields: CY (Current Yield) un YTC (Yield to Call)

Exercise 5.1

The nominal of bond is LVL 1000; it bears interest at 5% per annum, payable annually on 15. January. Its maturity time is 15 January 2009, and it is redeemed at par. Calculate clean and total price on 3 March 2005, if annual yield is 4%.

Example 5.2

Mortgage bond has nominal LVL 10000; it bears interest at 6% per annum, payable quarterly on 15 February, 15 May, 15 August and 15 November. Its maturity time is 15 May 2020 and it is redeemed at par. Calculate total price on 20 September 2005, if annual yield is 7%.

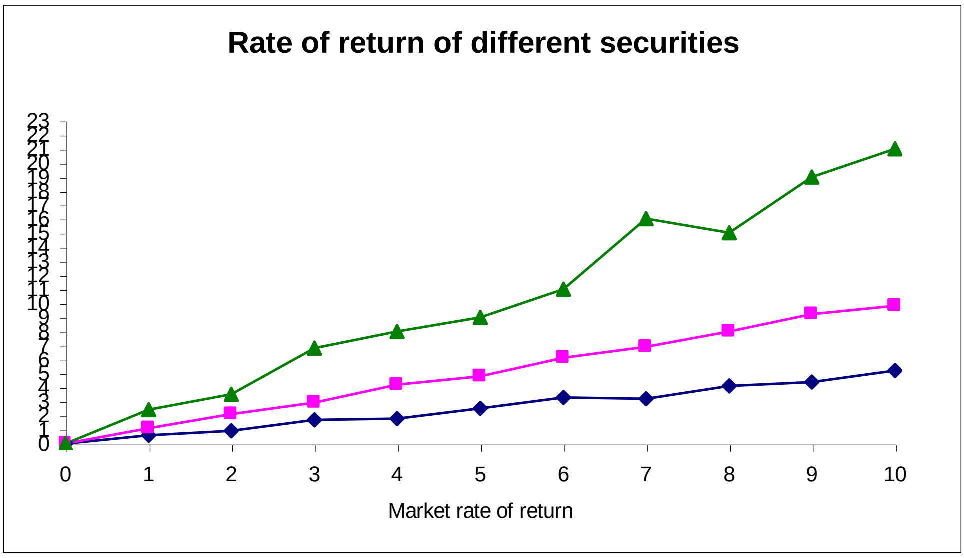
Example 5.3

Bond has nominal LVL 10000; it bears interest at 5% per annum, payable twice per year and it is redeemed at par after 5 years from next coupon payment. Calculate clean and total price what investor can pay if he wishes to have 4% rate of return per annum and 15 days have passed from previous coupon payment (167 left until next payment). Calculate YTM price of bond is LVL 10050.

6 STOCK VALUATIONS

Beta as measure of risk:

Market rate of return is rate of return of all traded asstes.



$$\hat{y} = a + b \cdot x \quad \text{where } b\text{=}slope,\ a\text{=}intercept$$

where x is market rate of return, y - corresponding security rate of return.

Example 6.1

Calculate beta of given security.

Market rate of return (%)	Security rate of return (%)
x	y
22	18
23	20
24	22
25	26
26	28

Beta is measure of market risk.

Beta	Direction of changes	Speed of changes
2	Direction of changes of security rate of return coincides with direction of changes of market rate of return.	Twice quicker.
1		The same.
0.5		Twice slowly
0	Changes do not have relationship.	
-0.5	Direction of changes of security rate of return is opposite to direction of changes of market rate of return.	Twice slowly.
-1		The same
-2		Twice quicker.

CAPM (Capital Asset Pricing Model)

Necessary rate of return on asset is rate of return what would be necessary with its risk measured with beta.

Risk-free rate of return is rate of return earned with assets without risk.

$$r_n = r_f + \beta \cdot (r_M - r_f)$$

where r_n - necessary rate of return, r_f - riskless rate of return, r_M - market rate of return.

Example 6.2

A) Given that the beta of security is 1.25, market rate of return 10% and risk free rate of return 6%, calculate necessary rate of return for given security.

B) Market rate of return is 13%, risk free rate of return is 8% and beta for different securities are given in table. Calculate necessary rate of return for given securities.

Security	Beta	Necessary rate of return
A	1,4	
B	0,8	
C	-0,9	

Used notations:

D_0 -divided paid in current year,

g - dividend growth rate,

P - present value of payment stream,

P_0 - current price of stock,

P_k - price of stock after k years (estimated),

k -time in years when investor is planning to sell stock.

Example 6.3

Market rate of return and corresponding rate of return on stock A is shown in following table. Assuming that market rate of return is planning for next year 18%, risk free rate of return is 4%, just paid dividend is LVL 0.8 and it is planned that dividends of that company will grow by annual rate 0.5 %, calculate 1) beta, 2) necessary rate of return on stock, 3) present value of future payments, 4) yield if current price of stock is LVL 5.5. To know that investor is planning to buy stock above current price and to sell it after 3 years above LVL 6, calculate internal rate of that transaction.

Market rate of return (%)	Stock rate of return (%)
17	12
20	15
18	15
21	16
23	17

Exercise 6.4

Market rate of return and corresponding rate of return on stock A is shown in following table. Assuming that market rate of return is planning for next year 22%, risk free rate of return is 6%, just paid dividend is LVL 1.8 and grows of dividends are not planning, calculate 1) beta, 2) necessary rate of return on stock, 3) present value of future payments, 4) yield if current price of stock is LVL 7. To know that investor is planning to buy stock above current price and to sell it after 5 years above LVL 12, calculate internal rate of that transaction.

Market rate of return (%)	Rate of return on stock (%)
13	12
18	15
18	15
20	16
22	17